

# Monopole Inflation in Brans-Dicke Theory

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## Abstract

According to previous work, topological defects expand exponentially without an end if the vacuum expectation value of the Higgs field is of the order of the Planck mass. We extend the study of inflating topological defects to the Brans-Dicke gravity. With the help of numerical simulation we investigate the dynamics and spacetime structure of a global monopole. Contrary to the case of the Einstein gravity, any inflating monopole eventually shrinks and takes a stable configuration. We also discuss cosmological constraints on the model parameters.

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## I. INTRODUCTION

For the last decade spacetime solutions of gravitating monopoles has been intensively studied in the literature [1–9]. This originated from rather mathematical interest in static monopole solutions. In both cases of global monopoles [1,2] and of magnetic monopoles [3], static regular solutions are nonexistent if the vacuum expectation value (VEV) of the Higgs field  $\eta$  is larger than a critical value  $\eta_{\text{sta}}$ , which is of the order of the Planck mass,  $m_{\text{Pl}}$ . The properties of monopoles for  $\eta > \eta_{\text{sta}}$  had been a puzzle for some time.

In connection with the above issue, it has been claimed by several authors independently that monopoles expand exponentially if  $\eta > O(m_{\text{Pl}})$  [4–6]. Among them, Guendelman and Rabinowitz [4] assumed the simplified model of a global monopole under the thin-wall approximation, while Linde [5] and Vilenkin [6] made qualitative arguments on any kinds of topological defects and argued that their core would inflate if the VEV of the Higgs field is large. Later the full evolution equations for monopoles were numerically solved [7,8], supporting the previous arguments as a whole. Furthermore, those numerical analyses revealed an unexpected result that the critical value,  $\eta_{\text{inf}}$ , above which inflation occurs in the core, is larger than  $\eta_{\text{sta}}$ , and stable but nonstatic solutions exist for  $\eta_{\text{sta}} < \eta < \eta_{\text{inf}}$  [8]. For example, in the case of global monopoles, we find  $\eta_{\text{sta}} \cong 0.20m_{\text{Pl}}$  and  $\eta_{\text{inf}} \cong 0.33m_{\text{Pl}}$  [8]. Global spacetime structure of an inflating monopole has also been discussed in [7–9].

In this paper, we extend the study of monopole inflation to the Brans-Dicke (BD) theory. If the BD field exists, the inflationary universe may not expand exponentially but expand with a power law even in the presence of an effective cosmological constant [10,11]. For example, in extended inflation [10], this slower expansion was expected to solve the graceful exit problem of old inflation [12]. We thus expect that the BD field also affects the dynamics and global spacetime structure of inflating monopoles.

We would also like to discuss whether the present model can be a realistic cosmological model or not. Once inflation happens, the spatial gradients of the fields become negligibly small on the scale of the observed universe. Therefore, just as other inflationary models, we have only to care about quantum fluctuations. This issue resolves itself into the commonplace analysis of density perturbations in the inflationary universe. Here we adopt the result obtained by Starobinsky and Yokoyama [13], who derived the general expression of the density fluctuations in the BD theory, taking account of the isocurvature mode as well as the adiabatic mode.

The rest of the paper is composed of two independent analyses. In the first part (§2), we investigate the dynamics and global spacetime structure of an inflating global monopole. In the second part (§3), we discuss cosmological constraints on this model, which come mainly from density perturbations. We use the units  $c = \hbar = 1$  throughout the paper.

## II. DYNAMICS AND SPACETIME STRUCTURE OF A GLOBAL MONOPOLE

The Brans-Dicke-Higgs system, which we consider here, is described by the action

$$S = \int d^4x \sqrt{-g} \left[ \frac{\Phi}{16\pi} \mathcal{R} - \frac{\omega}{16\pi\Phi} (\nabla_\mu \Phi)^2 - \frac{1}{2} (\nabla_\mu \Psi^a)^2 - V(\Psi) \right], \quad (2.1)$$

with

$$V(\Psi) = \frac{\lambda}{4}(\Psi^2 - \eta^2)^2, \quad \Psi \equiv \sqrt{\Psi^a \Psi^a}, \quad (2.2)$$

where  $\Phi$  and  $\Psi^a$  are the BD field and the real triplet Higgs field, respectively.  $\omega$  and  $\lambda$  are the BD parameter and the Higgs self-coupling constant, respectively.

Let us begin with a discussion of the fate of an inflating topological defect. Once inflation begins, the core region can be approximated by the flat Friedmann-Robertson-Walker spacetime:

$$ds^2 = -dt^2 + a(t)^2 d\mathbf{x}^2, \quad (2.3)$$

which yields the solution of extended inflation [14,10]:

$$a(t) \propto \left(1 + \frac{H_i t}{\alpha}\right)^{\omega + \frac{1}{2}}, \quad \Phi(t) \propto \left(1 + \frac{H_i t}{\alpha}\right)^2, \quad (2.4)$$

where  $H_i \equiv \sqrt{8\pi V(0)/3\Phi_i}$  is the Hubble parameter at the beginning of inflation, and  $\alpha^2 \equiv (2\omega + 3)(6\omega + 5)/12$ . Hence, the effective Planck mass,

$$m_{\text{Pl}}(\Phi) \equiv \sqrt{\Phi} \propto a^{\frac{1}{\omega + \frac{1}{2}}}, \quad (2.5)$$

continues to increase until inflation ends. This implies that, even if  $\eta/m_{\text{Pl}}(\Phi)$  is large enough to start inflation initially, it eventually becomes smaller than the critical value ( $\cong 0.33$ ) [7]. We thus speculate that any defect eventually shrinks after inflation.

In order to ascertain the above argument, we carry out numerical analysis for spherical global monopoles. The coordinate system we adopt is

$$ds^2 = -dt^2 + A^2(t, r) dr^2 + B^2(t, r) r^2 (d\theta^2 + \sin^2 \theta d\varphi^2). \quad (2.6)$$

For the matter fields, we adopt the hedgehog ansatz:

$$\Psi^a = \Psi(t, r) \hat{r}^a, \quad \hat{r}^a \equiv (\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta). \quad (2.7)$$

For the initial configurations of the Higgs fields and the BD field, we assume

$$\Psi(t = 0, r) = \eta \tanh\left(\frac{r}{\delta}\right) \quad \text{with} \quad \delta \equiv \frac{\sqrt{2}}{\sqrt{\lambda}\eta}, \quad \dot{\Psi}(t = 0, r) = 0, \quad (2.8)$$

$$\Phi(t = 0, r) = \Phi_i = \text{const.}, \quad \dot{\Phi}(t = 0, r) = 0, \quad (2.9)$$

where an overdot denotes  $\partial/\partial t$ .

Although the numerical code developed by one of us [8] worked well for monopoles in the Einstein theory, for the present system which includes the BD field it sometimes does not keep good accuracy for sufficient time. We therefore improve our numerical method, mainly based on the idea of Nakamura *et al.* [15]. The basic equations and our improved numerical method are presented in Appendix.

To illustrate our results, we take  $\omega = 1$ ,  $\lambda = 0.1$  and  $\eta/m_{\text{Pl}}(\Phi_i) = 1$ . The evolutions of  $\Psi$  and  $\eta/m_{\text{Pl}}(\Phi)$  are shown in Figure 1. Clearly the above arguments are verified by this

numerical result: as  $\eta/m_{\text{Pl}}(\Phi)$  gets close to the critical value, the monopole stops expanding and turns to shrink. In Fig. 2 we plot the trajectories of the position of  $\Psi = \eta/2$  for several values of  $\omega$ . The curves indicate that a monopole do not shrink toward the origin but tends to take a stable configuration.

Next, we shall examine the metric outside the monopole. Before we consider monopoles in the BD theory, let us review previous work on the outer solution in the Einstein theory briefly. The approximate expression for the metric outside a static monopole was found by Barriola and Vilenkin (BV) [1]:

$$ds^2 = - \left( 1 - \Delta - \frac{2M_c}{m_{\text{Pl}}^2 R} \right) dT^2 + \left( 1 - \Delta - \frac{2M_c}{m_{\text{Pl}}^2 R} \right)^{-1} dR^2 + R^2(d\theta^2 + \sin^2 \theta d\varphi^2). \quad (2.10)$$

where  $M_c$  is an integration constant and  $\Delta \equiv 8\pi\eta^2/m_{\text{Pl}}^2$ . Later Harari and Lousto pointed out that the mass parameter  $M_c$  is negative and therefore a global monopole has a repulsive nature [2]. For large  $R$ , the metric (2.10) can be approximated by

$$ds^2 = - (1 - \Delta) dT^2 + (1 - \Delta)^{-1} dR^2 + R^2(d\theta^2 + \sin^2 \theta d\varphi^2), \quad (2.11)$$

and therefore  $4\pi\Delta$  is interpreted as a deficit solid angle. Although the expression (2.11) looks singular when  $\Delta = 1$ , Cho & Vilenkin [9] showed that it is just an apparent singularity and (2.11) also applies to the case  $\Delta > 1$ .

Now we would like to discuss the properties of the outer spacetime by comparing it with the BV solution (2.10). As a coordinate-independent variable, we adopt the Misner-Sharp mass [16], which is defined as

$$\tilde{M} \equiv \frac{\bar{R}}{2} (1 - g^{\mu\nu} \bar{R}_{,\mu} \bar{R}_{,\nu}), \quad (2.12)$$

where  $\bar{R} \equiv \sqrt{g_{\theta\theta}}$  is the circumferential radius of the spacetime. Note that  $\tilde{M}$  has a dimension of length, and  $m_{\text{Pl}}^2 \tilde{M}$  is a mass in a usual sense. The BV solution is characterized by  $\tilde{M} = \tilde{M}_c + \Delta R$  with  $\tilde{M}_c = M_c/m_{\text{Pl}}^2$ .

Figure 3 shows a plot of  $\tilde{M}/\bar{R}$ . In the case of the Einstein theory (a),  $\tilde{M}/\bar{R}$  seems to converge into  $\Delta/2$  ( $=4\pi$  in this case) at large  $R$ , confirming that the metric is well approximated by (2.10). The result with the BD gravity looks similar, but a different character is that  $\tilde{M}/\bar{R}$  decreases with time even at a far region. This behavior is consistent with the BV solution in the Einstein theory: if we take account of the time-variation of  $m_{\text{Pl}}$  in the BV expression (2.10), a deficit solid angle effectively decreases and (2.10) also approximates the solution in the BD theory.

Our results obtained so far are rephrased as follows: our numerical integration of the evolution equations verifies as a whole the arguments with considering only the time-variation of  $m_{\text{Pl}}(\Phi)$ . That is, the metric in the core of a monopole is well approximated by (2.4), and the metric outside the core by (2.10) with changing  $m_{\text{Pl}}$ . Although we have chosen small  $\omega$  to illustrate the effect of the BD field clearly, if we take  $\omega > 500$  as constrained by observation [17], the spatial gradient of  $\Phi$  is much smaller, justifying more the arguments with neglecting the spatial configuration of  $\Phi$ .

The result that inflation eventually ends stems merely from the change of the local values of  $m_{\text{Pl}}(\Phi)$ . We may therefore extend this result to models of other topological defects.

### III. COSMOLOGICAL CONSTRAINTS

In this section we discuss constraints for this model to be a realistic cosmological model. As we mentioned in §1, the main constraint is from density perturbations. In the present model matter fields are composed of the BD field  $\Phi$  and the Higgs field  $\Psi^a$ . For the Higgs field, we only calculate fluctuations in the radial direction  $\delta\Psi$ , because fluctuations along the other directions do not contribute to growing modes.

Here we analyze the field equations in the Einstein frame by use of a conformal transformation, following previous work [11,13]. A wide class of generalized Einstein theories is described by the Einstein-Hilbert action with scalar fields:

$$\hat{\mathcal{S}} = \int d^4\hat{x} \sqrt{-\hat{g}} \left[ \frac{\hat{\mathcal{R}}}{2\kappa^2} - \frac{1}{2}(\hat{\nabla}\phi)^2 - \frac{e^{-\gamma\kappa\phi}}{2}(\hat{\nabla}\Psi)^2 - e^{-\beta\kappa\phi}V(\Psi) - U(\phi) \right] \quad \text{with } \kappa^2 \equiv 8\pi G, \quad (3.1)$$

where  $\beta$  and  $\gamma$  are constants, and  $\phi$  is the redefined scalar field.

In the BD theory, on which we concentrate in this section, the original action in the Jordan frame (2.1) is transformed into (3.1) with

$$\gamma = \frac{\beta}{2} = \sqrt{\frac{2}{2\omega + 3}}, \quad U(\phi) = 0 \quad (3.2)$$

through the conformal transformation and the redefinition of the scalar field

$$\hat{g}_{\mu\nu} = \frac{\Phi}{\Phi_0} g_{\mu\nu}, \quad \sqrt{\gamma}\kappa\phi \equiv \ln \frac{\Phi}{\Phi_0}, \quad (3.3)$$

with  $\Phi_0$  being a constant. From (3.3), the Planck mass in the Jordan frame is written as

$$m_{\text{Pl}}(\Phi) \equiv \sqrt{\Phi} = \sqrt{\Phi_0} e^{\gamma\kappa\phi/2}. \quad (3.4)$$

We set  $\phi = 0$  at the present time,  $t = t_0$ , identifying a constant  $\sqrt{\Phi_0}$  with the Planck mass at present. Because the evolution of the BD field after inflation is negligibly small for  $\gamma^2 \ll 1$ , we may identify  $\Phi$  at the end of inflation,  $\Phi_f$ , with  $\Phi_0$  (*i.e.*,  $\phi_f \cong \phi_0 = 0$ ). In the following we sometimes use the symbol  $m_{\text{Pl}}(\Phi)$  instead of  $\phi$ , and define  $m_{\text{Pl},0} \equiv m_{\text{Pl}}(\Phi_0) \cong m_{\text{Pl}}(\Phi_f)$ .

Hereafter, we omit a hat, which has denoted quantities in the Einstein frame. Taking the background as the spatially flat Friedmann-Robertson-Walker spacetime (2.3), the field equations for the homogeneous parts read

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{\kappa^2}{3} \left( \frac{1}{2} e^{-\gamma\kappa\phi} \dot{\Psi}^2 + \frac{1}{2} \dot{\phi}^2 + e^{-2\gamma\kappa\phi} V \right), \quad (3.5)$$

$$\ddot{\Psi} + 3H\dot{\Psi} - 2\gamma^2 \dot{\phi} \dot{\Psi} + e^{-\gamma\kappa\phi} \frac{dV}{d\Psi} = 0, \quad (3.6)$$

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\gamma\kappa}{2} e^{-\gamma\kappa\phi} \dot{\Psi}^2 - 2\gamma\kappa e^{-2\gamma\kappa\phi} V = 0. \quad (3.7)$$

The conditions for slow-roll inflation ( $|\ddot{\Psi}| \ll |3H\dot{\Psi}|$ ,  $|\ddot{\phi}| \ll |3H\dot{\phi}|$ , etc.) are equivalent to

$$\gamma^2 \ll \frac{3}{2}, \quad \left| \frac{m_{\text{Pl}}(\Phi)V_{,\Psi}}{V} \right| \ll \sqrt{48\pi}, \quad \left| \frac{m_{\text{Pl}}(\Phi)^2 V_{,\Psi\Psi}}{V} \right| \ll 24\pi, \quad (3.8)$$

where  $V_{,\Psi} \equiv dV/d\Psi$ . For the potential (2.2), the second and the third inequalities in (3.8) in the range  $0 < \Psi < \eta$  are equivalent to

$$\epsilon^{-1} \equiv \frac{\sqrt{6\pi}\eta}{m_{\text{Pl}}(\Phi)} \gg 1, \quad \Psi \ll \eta \left(1 - \frac{\epsilon}{2}\right) \equiv \Psi_f, \quad (3.9)$$

where terms of higher-order of  $\epsilon$  have been neglected in the second inequality.

One of the distinguished features of this model is that there are two scenarios of exiting from an inflationary phase and entering a reheating phase. This is illustrated by the slow-roll conditions (3.9). When the condition  $\epsilon^{-1} \gg 1$  (a more precise condition is  $\eta/m_{\text{Pl}}(\Phi) > 0.33$ ) breaks down, inflation stops globally. Before this time, many local regions with the present-horizon size enters a reheating phase when the second condition  $\Psi \ll \Psi_f$  breaks down. The second scenario is just like that of standard topological inflation or other slow-roll inflationary models. In the first scenario a microscopic monopole might remain in the observable universe. Unfortunately, however, this possibility turns out to be ruled out because the spectrum of density fluctuation is tilted excessively if  $\eta$  is too close to the critical value  $0.33m_{\text{Pl}}$  [18] and because  $\eta/m_{\text{Pl}}(\Phi_f)$  must be larger than 1 from the COBE normalization as will be seen later in Fig. 4. Thus we only consider the second (standard) reheating scenario below.

When inequalities (3.9) are satisfied, the field equations are approximated by

$$H^2 = \frac{\kappa^2}{3} e^{-2\gamma\kappa\phi} V, \quad (3.10)$$

$$3H\dot{\Psi} = -e^{-\gamma\kappa\phi} \frac{dV}{d\Psi}, \quad (3.11)$$

$$3H\dot{\phi} = 2\gamma\kappa e^{-2\gamma\kappa\phi} V, \quad (3.12)$$

which yield solutions [11,13],

$$N \equiv \ln\left(\frac{a_f}{a}\right) = -\frac{\kappa\phi}{2\gamma}, \quad (3.13)$$

$$N \cong \frac{1 - e^{-2\gamma^2 N}}{2\gamma^2} = \kappa^2 \int_{\Psi_f}^{\Psi} \frac{V}{V_{,\Psi}} d\Psi = \frac{\pi}{m_{\text{Pl},0}^2} \left( 2\eta^2 \ln \frac{\Psi_f}{\Psi} + \Psi^2 - \Psi_f^2 \right). \quad (3.14)$$

Starobinsky and Yokoyama [13] derived the amplitude of density perturbation on comoving scale  $l = 2\pi/k$  in terms of Bardeen's variable  $\Phi_A$  [19] as

$$\Phi_A(l)^2 = \frac{48V}{25m_{\text{Pl}}(\Phi)^4} \left[ \frac{8\pi V^2}{m_{\text{Pl}}(\Phi)^2 V_{,\Psi}^2} + \frac{(e^{2\gamma^2 N} - 1)^2}{4\gamma^2} \right], \quad (3.15)$$

where all quantities are defined at the time  $t_k$  when  $k$ -mode leaves the Hubble horizon, *i.e.*, when  $k = aH$ . Since the large-angular-scale anisotropy of the microwave background due to the Sachs-Wolfe effect is given by  $\delta T/T = \Phi_A/3$ , we can constrain the values of  $\lambda$  and  $\eta/m_{\text{Pl}}$  by the COBE-DMR data normalization [20],

$$\frac{1}{3}\Phi_A \cong 10^{-5}, \quad (3.16)$$

on the relevant scale. To calculate (3.16) explicitly, we have to relate scales during inflation and present scales. The number of  $e$ -folds  $N$  between  $t = t_k$  and  $t = t_f$  is given by

$$N_k = (1 - \gamma^2)^{-1} \left( 68 - \ln \frac{k}{a_0 H_0} + \ln \frac{V_f^{\frac{1}{4}}}{m_{\text{Pl},0}} + 2 \ln \frac{V_k^{\frac{1}{4}}}{V_f^{\frac{1}{4}}} - \frac{1}{3} \ln \frac{V_f^{\frac{1}{4}}}{\rho_{\text{rh}}^{\frac{1}{4}}} \right), \quad (3.17)$$

where  $\rho_{\text{rh}}$  is the energy density when reheating completes, which is not determined without specifying a reheating model. Here we choose  $N_{k=a_0 H_0} = 65$  typically. Then the corresponding values of  $\phi$  and  $\Psi$  are determined from (3.13) and (3.14). Figure 4 shows the allowed values of  $\lambda$  and  $\eta/m_{\text{Pl}}$  for  $\gamma = 0.045$  ( $\omega = 500$ ). The concordant values are represented by two curves, which is a distinguished feature for the double-well potential (2.2). Unfortunately, fine-tuning of  $\lambda$  ( $\lesssim 10^{-13}$ ) is needed, just like other models [13]. The condition of  $\lambda \lesssim 10^{-13}$  in the present potential was also found in [21]. The constraints for  $\omega > 500$  are practically no different from those in the Einstein gravity.

Using the relation  $d \ln k = da/a + dH/H$  and (3.10)-(3.12), we obtain

$$n - 1 \equiv \frac{d \ln \Phi_A^2}{d \ln k} \cong -\frac{3m_{\text{Pl}}(\Phi)^2 V_{,\Psi}^2}{8\pi V^2} + \frac{m_{\text{Pl}}(\Phi)^2 V_{,\Psi\Psi}}{4\pi V} - 6\gamma^2. \quad (3.18)$$

The spectral indices are plotted in Fig. 5; the two lines correspond to the two lines of COBE-normalized amplitudes in Fig. 4. As (3.18) suggests, the deviation from the Einstein theory is only  $6\gamma^2 \cong 0.01$  for  $\omega = 500$ . Therefore, relatively large shift from  $n = 1$  is caused not by the BD field but by the double well potential (2.2).

Besides the amplitude of density perturbation, we have to check other conditions for successful inflation. First, if we take quantum fluctuations  $\delta\Psi_Q$  into account, the classical dynamics is meaningless unless

$$\frac{|\dot{\Psi}|}{H} > \delta\Psi_Q = \frac{e^{\gamma\kappa\phi/2} H}{2\pi}. \quad (3.19)$$

We check the value of  $\dot{\Psi}$  at  $k = a_0 H_0$  which satisfies the COBE normalization, finding that (3.19) is always satisfied.

The second is the condition that classical description of spacetime is valid. We examine this because we have considered the case of  $\eta \cong m_{\text{Pl}}$ , which may be critical. We require

$$V(\Psi_i) < m_{\text{Pl}}(\Phi_i)^4. \quad (3.20)$$

This condition is also satisfied for  $\eta \cong m_{\text{Pl},0}$  and  $\lambda \lesssim 10^{-13}$ . In the end the two conditions do not add new constraints on the model.

Now, let us discuss the detectability of relic monopoles in this model. First, we investigate how the long-range term of the monopole mass density,  $\rho_{\text{gm}}(R) = \eta^2/R^2$ , contributes to the cosmological mass density, where  $R$  is the distance from a monopole core to an observer. Hiscock [22] derived the condition that the averaged mass density of multiple global monopoles does not exceed the cosmological mass density, *i.e.*,  $\langle \rho_{\text{gm}} \rangle < \rho_0$ . His result is equivalent to

$$\frac{N}{V_H} < 5 \times 10^{-4} \Omega_0^{\frac{2}{3}} \left( \frac{m_{\text{Pl},0}}{\eta} \right), \quad (3.21)$$

where  $N/V_H$  is the number of monopoles in the the present horizon volume,  $V_H \equiv (4\pi/3)H_0^{-3}$ , and  $\Omega_0$  is the present density parameter. This indicates that the existence of even one monopole is critical for  $\eta \cong m_{\text{Pl},0}$ . To see it more closely, we consider the condition that the local density by one monopole does not exceed the cosmological mass density, *i.e.*,  $\rho_{\text{gm}}(R) < \rho_0$ . This leads

$$R > \frac{3H_0^{-1}}{\sqrt{\Omega_0}} \left( \frac{\eta}{m_{\text{Pl},0}} \right), \quad (3.22)$$

which shows that the center of a monopole should be located farther than the horizon. A more stringent constraint is obtained by considering the difference of  $\rho_{\text{gm}}$  at both ends of the observed universe:

$$\Delta\rho_{\text{gm}} = \frac{\eta^2}{(R - H_0^{-1})^2} - \frac{\eta^2}{(R + H_0^{-1})^2} \cong \frac{4\eta^2 H_0^{-1}}{R^3}. \quad (3.23)$$

The condition  $\Delta\rho_{\text{gm}}/\rho_0 < 10^{-5}$  leads

$$R > \frac{150H_0^{-1}}{\Omega_0^{\frac{1}{3}}} \left( \frac{\eta}{m_{\text{Pl},0}} \right)^{\frac{2}{3}}. \quad (3.24)$$

We now argue the plausible value of  $R$  realized in this model. By assumption our universe was once in the core of a monopole with  $\Psi \cong 0$  and the Higgs field started classical evolution only after it had acquired an amplitude  $\Psi \gtrsim \delta\Psi_Q$ . The number of  $e$ -folds after this epoch is approximately given by

$$N = 2\pi \left( \frac{\eta}{m_{\text{Pl},0}} \right)^2 \ln \left( \sqrt{\frac{6\pi}{\lambda}} \frac{m_{\text{Pl},0}}{\eta} \right). \quad (3.25)$$

For  $\eta = m_{\text{Pl},0}$  and  $\lambda = 10^{-13}$ , we find  $N = 103$ . That is, the initial core radius becomes  $e^{103-65} \sim 10^{16}$  times as large as the present horizon. We therefore conclude that the core of a monopole is located so far from the observed region that even its long-range term  $\rho_{\text{gm}} = \eta^2/R^2$  does not exert effective astrophysical influence. Because this conclusion is irrelevant to the long-range term, it suggests that a local monopole in the same model is not detectable either.



#### IV. SUMMARY AND DISCUSSION

We have considered inflating monopoles in the BD theory. First, we have investigated the dynamics and spacetime structure of a global monopole. In the Einstein theory, monopoles undergo eternal inflation if and only if  $\eta/m_{\text{Pl}} \gtrsim 0.33$ . In the BD theory, on the other hand, the ratio  $\eta/m_{\text{Pl}}(\Phi)$  decreases until inflation ends. That is, if  $\eta/m_{\text{Pl}}(\Phi)$  is large enough initially, a monopole starts to expand; however, after the ratio becomes smaller than the critical value ( $\cong 0.33$ ), the monopole turns to shrink. This behavior is quite different from that in the Einstein gravity and gives another mechanism to terminate inflation, which, however, is irrelevant to our universe.

Secondly, we have examined cosmological constraints on the parameters, which stems mainly from density perturbations. The COBE normalization requires  $\eta/m_{\text{Pl},0} \gtrsim 1$ ,  $\lambda \lesssim 10^{-13}$  for  $\omega > 500$ , implying that inflation in our universe must have ended in the standard manner as  $\Psi$  reaches  $\eta$ . We also calculate the spectral index, which turns out to be typically around  $n \cong 0.9$ .

Although we have concentrated on the BD theory, finally we make a brief discussion on the behavior in general theories (3.1). We still assume  $U(\phi) = 0$ . The slow-roll conditions for the models are

$$\frac{\beta^2}{2} \text{ and } |\beta\gamma| \ll 3, \quad \frac{e^{\gamma\kappa\phi/2}}{\kappa} \left| \frac{V_{,\Psi}}{V} \right| \ll \sqrt{6} \text{ and } \sqrt{\left| \frac{6\beta}{\gamma} \right|} (\gamma \neq 0), \quad \frac{e^{\gamma\kappa\phi}}{\kappa^2} \left| \frac{V_{,\Psi\Psi}}{V} \right| \ll 3 \quad (4.1)$$

and the behavior of  $\phi$  is given by

$$\phi - \phi_i = \frac{\beta}{\kappa} \ln \frac{a}{a_i}. \quad (4.2)$$

These show that, if  $\beta\gamma > 0$ ,  $\gamma\phi$  increases and the slow-roll conditions eventually break down. We may therefore conclude that the finite duration of topological inflation is a general nature in generalized Einstein theories.

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## APPENDIX A: FIELD EQUATIONS AND NUMERICAL METHOD

In this Appendix we explain how we solve the field equations for a global monopole. As we shall show below, we improve our previous schemes [7] so as to keep better accuracy.

The variation of (2.1) with respect to  $g_{\mu\nu}$ ,  $\Phi^a$ , and  $\Phi$  yield the field equations:

$$G_{\mu\nu} \equiv \mathcal{R}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\mathcal{R} = \frac{8\pi}{\Phi}T_{\mu\nu} + \frac{\omega}{\Phi} \left[ \nabla_\mu \Phi \nabla_\nu \Phi - \frac{1}{2}g_{\mu\nu}(\nabla\Phi)^2 \right] + \nabla_\mu \nabla_\nu \Phi - g_{\mu\nu}\square\Phi, \quad (\text{A1})$$

$$\square\Psi^a = \frac{\partial V(\Psi)}{\partial\Psi^a}, \quad (\text{A2})$$

$$\square\Phi = \frac{8\pi T_\sigma^\sigma}{2\omega + 3}, \quad (\text{A3})$$

with

$$T_{\mu\nu} \equiv \nabla_\mu \Psi^a \nabla_\nu \Psi^a - g_{\mu\nu} \left[ \frac{1}{2}(\nabla\Psi^a)^2 + V(\Psi) \right]. \quad (\text{A4})$$

In the following we shall write down the field equations (A1), (A2), and (A3) with the metric (2.6) and the hedgehog ansatz (2.7). To begin with, we introduce the extrinsic curvature tensor  $K_{ij}$ :

$$K_r^r = -\frac{\dot{A}}{A}, \quad K_\theta^\theta (= K_\varphi^\varphi) = -\frac{\dot{B}}{B}, \quad K \equiv K_i^i. \quad (\text{A5})$$

Next, following Nakamura *et al.* [15], we define the following variables to guarantee the regularity conditions at the center:

$$a \equiv \frac{A - B}{r^2}, \quad k \equiv \frac{K_\theta^\theta - K_r^r}{r^2}, \quad (\text{A6})$$

and introduce a new space variable,  $x \equiv r^2$ . Because only  $\Psi$  is an odd function of  $r$  due to the hedgehog ansatz, we adopt a variable  $\psi \equiv \Psi/r$  instead of  $\Psi$ .

There are several advantages in the above procedure of Nakamura *et al.* [15] The first is that any diverging factor at  $r = 0$  like  $1/r$  does not appear in the basic equations below; hence we do not have to treat the  $r = 0$  point separately. The second is that the new variables  $a$  and  $k$  are replaced with differences between comparable quantities like  $A - B$  at  $r \approx 0$ , which would generate numerical errors due to a finite decimal of computers. The third is that a derivative at  $r \approx 0$  is approximated by a finite difference much more accurately. To see this, let us imagine a function  $F(r)$  which is expanded as

$$F(r) = c_0 + c_1 r + c_2 r^2 + c_3 r^3 + \dots, \quad (\text{A7})$$

around  $r = 0$ . If we take the central difference of  $F$  with respect to  $r$ , its derivative at  $r = \Delta r$  is approximated as

$$\frac{dF}{dr}(\Delta r) = \frac{F(2\Delta r) - F(0)}{2\Delta r} + O(\Delta r^2), \quad (\text{A8})$$

that is, it is approximated only up to the  $c_2$ -term. If  $F$  is given as an even or odd function of  $r$  and if we use a space variable  $x$ , on the other hand, we can expand it as

$$F_{\text{even}} = c_0 + c_2x + c_4x^2 + \dots, \quad (\text{A9})$$

$$\frac{F_{\text{odd}}}{r} = c_1 + c_3x + c_5x^2 + \dots. \quad (\text{A10})$$

If we take the central difference with respect to  $x$  at  $x = (\Delta r)^2$ , for an even (odd) function, it approximates  $dF/dx$  up to  $c_4$  ( $c_5$ ), and moreover guarantees  $c_5 = 0$  ( $c_6 = 0$ ).

Furthermore, we define the following auxiliary variables to make the equations have a first-order form:

$$C \equiv -\frac{B'}{B}, \quad (\text{A11})$$

$$\varpi \equiv \dot{\psi}, \quad \xi \equiv \psi', \quad (\text{A12})$$

$$\Pi \equiv \dot{\Phi}, \quad \Xi \equiv \Phi', \quad (\text{A13})$$

where a prime and an overdot  $\partial/\partial x$  and  $\partial/\partial t$ , respectively. A full set of dynamical variables is  $a$ ,  $B$ ,  $C$ ,  $\psi$ ,  $\xi$ ,  $\Phi$ ,  $\Xi$ ,  $K$ ,  $k$ ,  $\varpi$ , and  $\Pi$ . The time derivatives the first seven variables are given by the definitions above:

$$\dot{a} = -aK_r^r + Bk, \quad (\text{A14})$$

$$\dot{B} = -BK_\theta^\theta, \quad \dot{C} = K_\theta^{\theta'}, \quad (\text{A15})$$

$$\dot{\psi} = \varpi, \quad \dot{\xi} = \varpi', \quad (\text{A16})$$

$$\dot{\Phi} = \Pi, \quad \dot{\Xi} = \Pi'. \quad (\text{A17})$$

Note that the value of  $K_\theta^{\theta'}$  in (A15) can be determined by the momentum constraint (A19), which gives a more accurate value than a finite difference of  $K_\theta^\theta$ .

Now we write down the field equations (A1), (A2) and (A3) as

$$\begin{aligned} -G_t^t &\equiv \frac{K^2 - k^2x^2}{3} + \frac{8x}{A^2} \left( C' - \frac{A'C}{A} - \frac{3C^2}{2} \right) + \frac{4}{A^2} \left[ \frac{A'}{A} + 4C + \frac{a(A+B)}{4B^2} \right] \\ &= \frac{8\pi}{\Phi} \left[ \frac{x\varpi^2}{2} + \frac{2x\xi}{A^2}(x\xi + \psi) + \frac{\psi^2}{2A^2} + \frac{\psi^2}{B^2} + V \right] \\ &\quad + \frac{\Pi}{\Phi} \left( \frac{\omega\Pi}{2\Phi} + K \right) + \frac{4}{A^2\Phi} \left[ \Xi' + x\Xi \left( \frac{\omega\Pi}{2\Phi} - \frac{A'}{A} - 2C \right) + \frac{3\Xi}{2} \right], \end{aligned} \quad (\text{A18})$$

$$\frac{G_{tr}}{4r} \equiv K_\theta^{\theta'} + k \left( \frac{1}{2} - xC \right) = \frac{2\pi\varpi}{\Phi}(2x\xi + \psi) + \frac{1}{2\Phi} \left[ \Pi' + \Xi \left( \frac{\omega\Pi}{\Phi} + K_r^r \right) \right], \quad (\text{A19})$$

$$\begin{aligned} -\mathcal{R}_t^t &\equiv \dot{K} - \frac{K^2 + 2x^2k^2}{3} = \frac{8\pi}{\Phi}(x\varpi^2 - V) - \frac{12\pi T_\sigma^\sigma}{(2\omega + 3)\Phi} \\ &\quad + \frac{\Pi}{\Phi} \left( \frac{\omega\Pi}{\Phi} + K \right) + \frac{4}{A^2\Phi} \left[ x\Xi' - x\Xi \left( \frac{A'}{A} + 2C \right) + \frac{3\Xi}{2} \right], \end{aligned} \quad (\text{A20})$$

$$\begin{aligned} \frac{\mathcal{R}_r^r - \mathcal{R}_\theta^\theta}{r^2} &\equiv \dot{k} - kK + \frac{2}{A^3} \left[ 2AC' - 2A'C + aC + a' - \frac{a^2}{B^2} \left( \frac{A}{2} + B \right) \right] \\ &= \frac{8\pi}{A^2\Phi} \left[ 4\xi(x\xi + \psi) - \frac{\psi^2 a(A+B)}{B^2} \right] - \frac{k\Pi}{\Phi} + \frac{4}{A^2\Phi} \left[ \Xi' + \Xi \left( \frac{\omega\Pi}{\Phi} - \frac{A'}{A} + C \right) \right], \end{aligned} \quad (\text{A21})$$

$$\dot{\varpi} = K\varpi + \frac{4}{A^2} \left[ x\xi' - x\xi \left( \frac{A'}{A} + 2C \right) + \frac{5\xi}{2} \right] - \frac{2\psi}{A^2} \left[ \frac{A'}{A} + 2C + \frac{a(A+B)}{B^2} \right] - \lambda\psi(\Psi^2 - \eta^2), \quad (\text{A22})$$

$$\dot{\Pi} = K\Pi + \frac{4}{A^2} \left[ x\Xi' - x\Xi \left( \frac{A'}{A} + 2C \right) + \frac{3\Xi}{2} \right] - \frac{8\pi T_\sigma^\sigma}{2\omega + 3}, \quad (\text{A23})$$

with

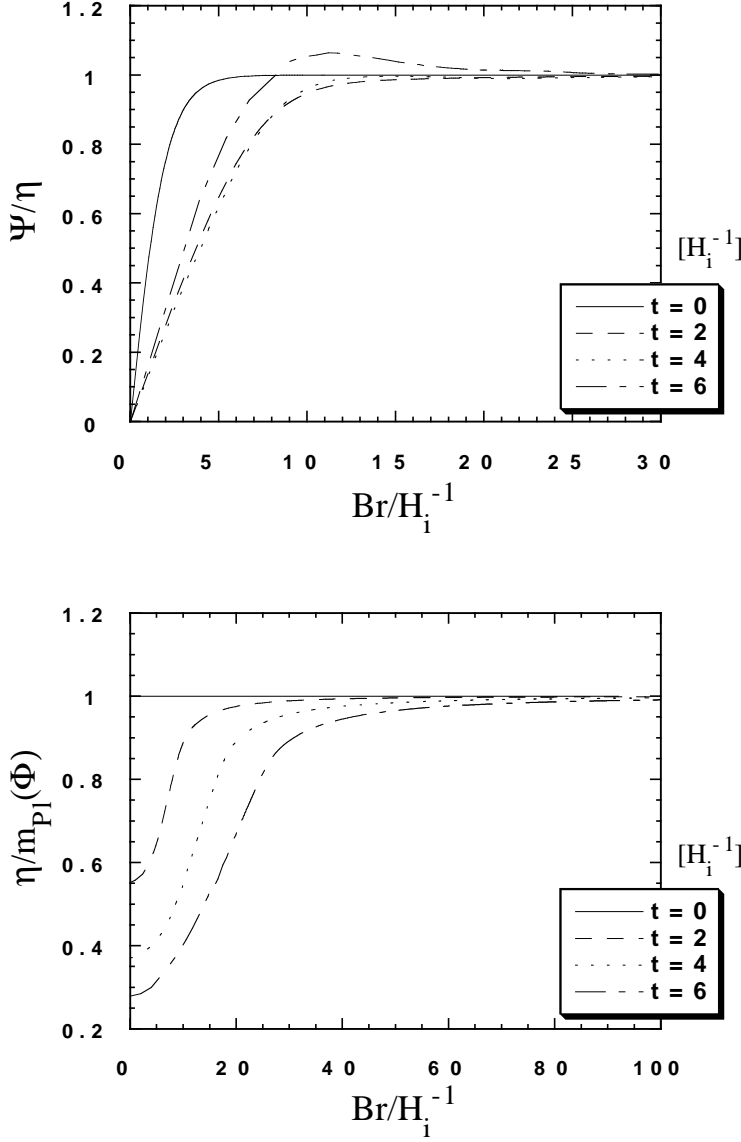
$$T_\sigma^\sigma \equiv x\varpi^2 - \frac{4x\xi}{A^2}(x\xi + \psi) - \psi^2 \left( \frac{1}{A^2} + \frac{2}{B^2} \right) - 4V. \quad (\text{A24})$$

In order to set up initial data, we assume  $A(t=0, r) = B(t=0, r) = 1$  besides the matter configurations (2.8) and (2.9), and solve the constraint equations (A18) and (A19) to determine  $K(t=0, r)$  and  $k(t=0, r)$ . Equation (A20), (A21), (A22), and (A23) provide the next time step of  $K$ ,  $k$ ,  $\varpi$ , and  $\Pi$ , respectively. The constraint equations (A18) and (A19) remain unsolved during the evolution and are used for checking the numerical accuracy. Through all the calculations the errors are always less than a few percent.

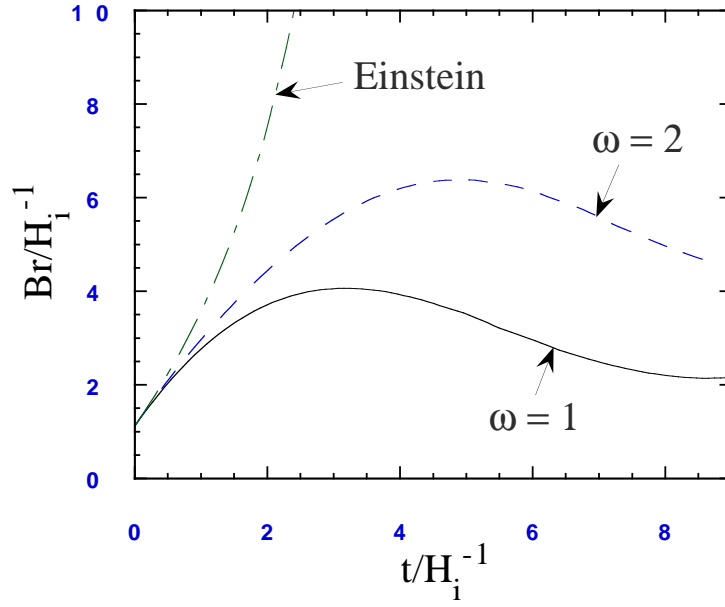
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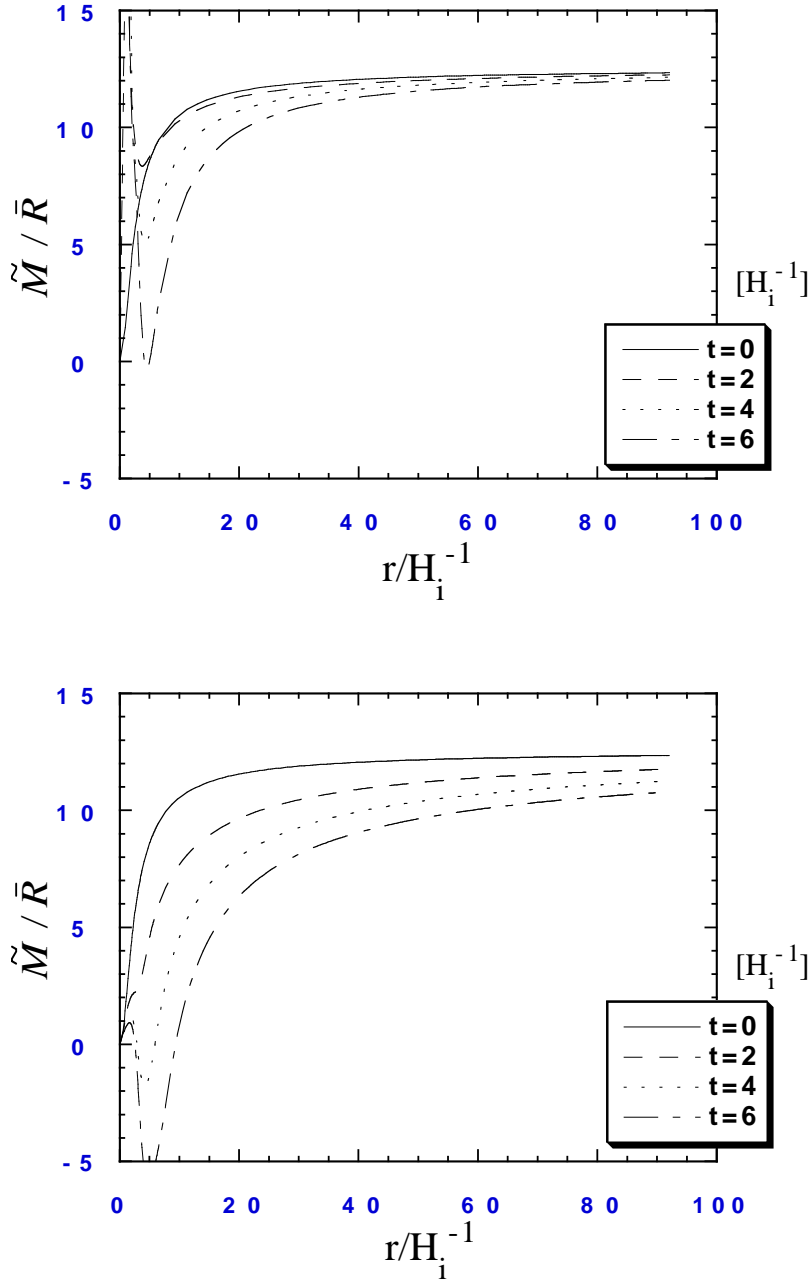
# FIGURES



**FIG.1.** Dynamics of a global monopole with  $\omega = 1$ ,  $\lambda = 0.1$ , and  $\eta/m_{\text{Pl}}(\Phi_i) = 1$ . We plot  $\Psi$  and  $\eta/m_{\text{Pl}}(\Phi)$  in (a) and in (b), respectively. Because we omit the far region where  $\Psi$  is almost constant in (a), the scales of abscissas in (a) and in (b) are different.

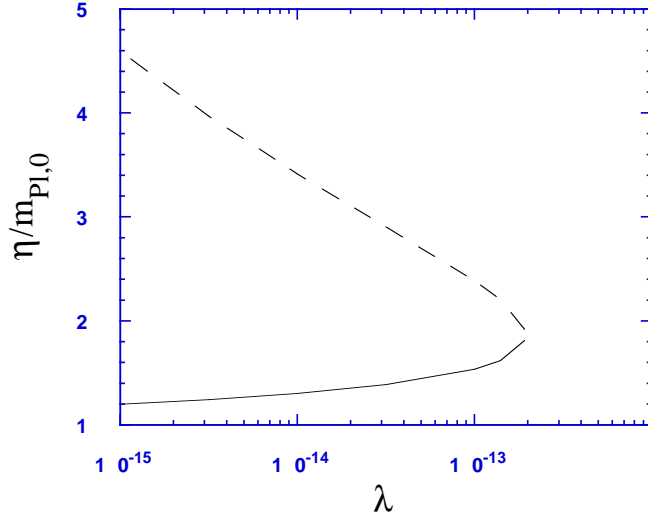


**FIG.2.** Dependence of the evolution of a global monopole on  $\omega$ . We set  $\lambda = 0.1$  and  $\eta/m_{\text{Pl}}(\Phi_i) = 1$ . We plot trajectories of the position of  $\Psi = \eta/2$ .

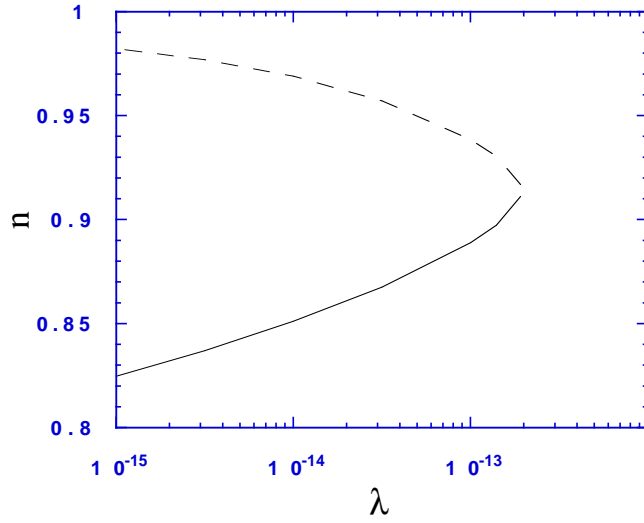


**FIG. 3.** Behavior of the Misner-Sharp mass outside the core of a monopole. We take, for reference, the Einstein gravity in (a), and  $\omega = 1$  in (b).





**FIG. 4.** Concordant values of  $\lambda$  and  $\eta/m_{Pl,0}$  with COBE-normalized amplitudes of density perturbations.



**FIG. 5.** The spectral indices of density perturbations. These two index curves correspond to the two amplitude curves in Fig. 4.